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Operational Research Project

Idea # 4

Service/Facility allocation

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Table of Contents

[1. Part 1 3](#_Toc195825921)

[1.1. Application Description 3](#_Toc195825922)

[1.2. Linear programming formulation 5](#_Toc195825923)

[Formulation 1: incapacitated Facility Location Problem (UFLP) 5](#_Toc195825924)

[Formulation 2: Capacitated Facility Location Problem (CFLP) 6](#_Toc195825925)

[Formulation 3: UFLP with Improved Approximation 7](#_Toc195825926)

[1.3. Step 3: Comparing the 3 formulations 8](#_Toc195825927)

[Formulation 1: incapacitated Facility Location Problem (UFLP): 8](#_Toc195825928)

[Formulation 2: Capacitated Facility Location Problem (CFLP): 8](#_Toc195825929)

[Formulation 3: UFLP with Improved Approximation Algorithm: 8](#_Toc195825930)

[1.4. Solving one model using LP 9](#_Toc195825931)

[1.4.1 Solving the LP model using Excel Solver 9](#_Toc195825932)

[1.4.3. Dual Formulation 11](#_Toc195825933)

[1.4.3. Sensitivity Report 12](#_Toc195825934)

[2. Part 2 – Personal Optimization Problem 14](#_Toc195825935)

[2.1. Problem Description 14](#_Toc195825936)

[2.2. Linear Programming 16](#_Toc195825937)

[2.3. Excel Solver Solution & Sensitivity Report 18](#_Toc195825938)

[2.4. Convert to Dual Formulation 20](#_Toc195825939)

[3. References 21](#_Toc195825940)

# Part 1

## Application Description

The service or facility allocation problem is a well-known topic in operations research. It focuses on choosing the best locations for service centers like hospitals, clinics, or data centers. The main goal is to serve a group of clients effectively while either reducing costs, shortening distances, or increasing coverage. These types of decisions are very important in real-life areas such as transportation (like bus stops and metro stations), healthcare (like clinics and hospitals), and emergency services (such as fire stations and police departments).

Linear Programming (LP) is a powerful and popular method used to model and solve these types of problems. It works well because of its simple mathematical structure and its ability to handle many constraints and objectives at the same time.

In this part of the project, I reviewed three research papers that used pure LP models. That means the decision variables in these models are continuous (not integers or binary). These types of models are especially useful for large systems or when fractional solutions are acceptable or can later be rounded into whole numbers.

The first paper I reviewed is titled "A Study for the Facility Location Problem" (2019), published in Procedia Manufacturing. This paper focuses on the Incapacitated Facility Location Problem (UFLP), which assumes that each facility can serve unlimited demand.   
The authors built an LP model where the variables show how much of each client’s demand goes to each facility. The goal is to reduce the total cost usually based on distance that the client will walk or cover or transportation fees. The constraints make sure all customer demand is fully met and that services only come from facilities that are “opened.” Although this model uses continuous variables for simplicity, the paper also argues how local search techniques can improve the results for real world applications.

The second paper is *"LP-Based Approximation Algorithms for Capacitated Facility Location"* (2004) by Chaitanya Swamy. This one looks at the Capacitated Facility Location Problem (CFLP), where each facility has a limit on how much it can operate. This reflects real world examples like hospitals with limited beds or delivery centers with limited staff. The model uses LP relaxation meaning the integer limitations are removed to allow fractional solutions. These fractional solutions are then used in approximation methods to create almost accurate final solutions. The objective of this model includes both the fixed cost of opening facilities and the cost of serving customers. Constraints make sure no facility go over its capacity and that all customer needs are met. Even though final decisions in real life would need to be integers, this paper shows that LP models can still be very useful for producing high quality results.

The third paper is *"Improved Approximation Algorithms for the incapacitated Facility Location Problem"* (2003) by Fabian Chudak and David Shmoys. Like the first paper, it also studies UFLP, but from a more theoretical angle. It introduces an LP model where customer assignments and facility decisions are continuous. What stands out in this paper is a better rounding method that changes the LP solution into a valid integer solution, while keeping the total cost very close to the original LP value. The authors show that this method performs better than older ones, and they back it up with strong mathematical results. Even though this LP model does not use binary variables, it still proves to be very effective as a starting point for solving real world problems.

## Linear programming formulation

### Formulation 1: incapacitated Facility Location Problem (UFLP)

(Based on: A Study for the Facility Location Problem, 2019)

Decision Variables:

: Amount of demand of customer j served by facility i

: Fraction of facility i opened (continuous between 0 and 1)

Objective Function:  
Minimize total transportation cost:

Constraints:

Demand satisfaction for each customer:

Facility must be open to serve:

Non-negativity and bounds:

### Formulation 2: Capacitated Facility Location Problem (CFLP)

(LP-Based Approximation Algorithms for CFLP, 2004)

**Decision Variables:**

: Demand of customer j served by facility i

: Fraction of facility i opened

: Fixed cost to open facility i

**Objective Function:**  
Minimize total cost (fixed + variable): Min Z=

**Constraints:**

Customer demand must be met:

Facility capacity constraint:

Non-negativity and bounds:

### Formulation 3: UFLP with Improved Approximation

(Based on: Improved Approximation Algorithms for UFLP, 2003)

**Decision Variables:**

: Fraction of demand of customer j assigned to facility i

: Usage level of facility i

: Fixed cost of using facility i

**Objective Function:**  
Minimize total cost:

**Constraints:**

Each customer must be fully served:

Assignment only allowed if facility is open:

Non-negativity and bounds:

## Step 3: Comparing the 3 formulations

### Formulation 1: incapacitated Facility Location Problem (UFLP):

* Does not consider capacity limits or fixed facility costs
* Simple and easy to solve for small to medium problems

### Formulation 2: Capacitated Facility Location Problem (CFLP):

* More realistic model
* Includes both service and fixed costs
* Suitable for large scale or real life facility problems

### Formulation 3: UFLP with Improved Approximation Algorithm:

* Uses LP relaxation and rounding methods
* Produces high quality near ideal solutions
* Mostly used in theoretical or algorithmic research

## Solving one model using LP

## 1.4.1 Solving the LP model using Excel Solver

Problem Setup (Hypothetical but Realistic Data):

* 4 facilities (i = 1, 2, 3, 4)
* 3 customers (j = 1, 2, 3)
* Each customer has a demand
* Each facility has a capacity
* We know the costs and fixed opening costs

Decision Variables:

* : amount of customer j’s demand served by facility i
* : 1 if facility i is opened

Objective Function:

Minimize total cost:

Z = f₁y₁ + f₂y₂ + f₃y₃ + f₄y₄ +

(c₁₁x₁₁ + c₁₂x₁₂ + c₁₃x₁₃ +

c₂₁x₂₁ + c₂₂x₂₂ + c₂₃x₂₃ +

c₃₁x₃₁ + c₃₂x₃₂ + c₃₃x₃₃ +

c₄₁x₄₁ + c₄₂x₄₂ + c₄₃x₄₃)

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## Dual Formulation

Let:

= dual variable for the demand constraint for customer j

= dual variable for the capacity constraint for facility i

Objective function:

Maximize:

Subject to constraints:

Variable conditions:

## Sensitivity Report

**A screenshot of a table

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After solving the Capacitated Facility Location Problem using excel solver. Then I generated the sensitivity report to understand better how changes in the models parameters affect the optimal solution.

Key Observations:

* The shadow price for Clients 1's demand constraint is 4, which means that if the demand increases by one unit, the total cost would increase by 4 as long as the change is within the allowable range.
* The reduced cost for x₁₃ is 6, showing that the unit shipping cost would need to decrease by 6 for this variable to become part of the best solution.
* The allowable increase and decrease values show how much each cost factor can change without altering the current optimal basis.
* Facilities 1 and 2 have zero reduced costs, confirming they are active (used) in the solution, while facilities 3 and 4 are not used and have positive reduced costs.

This analysis will help us understand and see how stable the current solution is and whether small changes in input value would affect which facilities to open or how demand should be served

# Part 2 – Personal Optimization Problem

## Problem Description

As a student that is obsessed with gaming I usually face challenges to manage my time between studying and relaxation. Gaming is my main source of entertainment , but with the large project specially in my major(CS). So I want to enjoy my gaming hours without feeling mentally drained. So when we took the linear programming in our operations research course I got an idea to treat my weekly gaming schedule as a linear programming problem.

I have a limited time of 20 hours available per week for gaming.

I usually play 3 games:

* War Thunder
  + A realistic military combat simulator
* Warzone
  + A fast-paced, intense battle royale shooter
* Tom Clancy's Rainbow Six Siege
  + A tactical FPS that requires strategy and communication

Each game provides a different level of enjoyment/hour and it depends which friends are playing with me so it varies.

* War Thunder
  + 5 enjoyment points/hour
* Warzone
  + 6 enjoyment points/hour
* Tom Clancy's Rainbow Six Siege
  + 4 enjoyment points/hour

But also each game we drain my mental effort and different levels of focus.

* War Thunder
  + 5 focus points/hour
* Warzone
  + 8 focus points/hour
* Tom Clancy's Rainbow Six Siege
  + 6 focus points/hour

And for my mental effort and to stay productive I must maintain a weekly focus limit of 100 points.

My personal preference are: at least 4 hours/ week of War Thunder to maintain my long term progression,

I don’t want to spend more than 8 hours on Warzone, to avoid fatigue.

I want to limits Rainbow Six Siege to maximum of 6 hours, since it require high level of concentration to be able to perform in the game.

Because the relationships are linear (each hour brings a fixed amount of enjoyment and uses a fixed amount of focus), LP is the right tool to solve this. By doing so, I can enjoy my favorite games without feeling guilty or exhausted, and still maintain a healthy balance between fun and focus.

## Linear Programming

Decision Variables:

Let:

Objective function:

War Thunder = 5 points/hour

Warzone = 6 points/hour

Rainbow Six Siege = 4 points/hour

Maximize Z=

Constraints:

Total time limit:

Total focus limit:

War Thunder = 5 points/hour

Warzone = 8 points/hour

Rainbow Six = 6 points/hour

Max focus = 100

Game Time limits:

Play War Thunder at least 4 hours:

Play Warzone no more than 8 hours:

Play Rainbow Six no more than 6 hours:

Non-negativity:

## Excel Solver Solution & Sensitivity Report

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A screenshot of a table

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## Convert to Dual Formulation

Let:

Minimize W =

For

For

For

Variables condition:

## References

First Source: A Study for the Facility Location Problem (2019).

<https://www.sciencedirect.com/science/article/pii/S2405896319315198>

Second Source: LP-Based Approximation Algorithms for Capacitated Facility Location.

<https://arxiv.org/abs/1407.3263>

Third Source: Improved Approximation Algorithms for the Uncapacitated Facility Location Problem (2003).

https://www.researchgate.net/publication/220618395\_Improved\_Approximation\_Algorithms\_for\_the\_Uncapacitated\_Facility\_Location\_Problem